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Existential Graphs and Thirdness[1]

Robert Burch has, in **A Peircean Reduction Thesis**, made a major contribution to communication

between what could be called Peircean logic and the more "traditional" approach based on the work of

Frege, Peano, and Russell; we may call this the **FPR** approach. And in so contributing, he helps bridge

gaps in understanding which may stand between those focusing on the semiotic, and those more

committed to that "traditional" approach. Others (than Frege, Peano, and Russell), of course, have

contributed significantly to that tradition--the names I list here are intended to be guideposts to an

approach to mathematical logic which differs in certain major respects from that of Peirce (see Zeman

1986). Burch provides an algebraic translation (his "PAL") of Peirce's existential graphs which enables

certain Peircean results concerning the reducibility of all relations to triadic relations to receive a

formulation in terms of the **FPR** approach; as Burch notes,

> Charles S. Peirce repeatedly maintained that relations of adicity higher than
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> three[2] could be reduced to relations of adicity three or less, while relations of the first
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> three adicities could not in general be reduced. This claim has seemed to many
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> philosophers to be bizarre in light of various twentieth-century results in logic [rooted in
>
> the work of the **FPR** school!] that show that all relations can be reduced to dyadic ones
>
> (Burch 1991, vii).

But in addition to laying out this link, Burch's book underlines some themes which are of considerable

import in understanding Peirce's work, and, in its algebraic approach, provides us with interesting

diagrammatic tools to aid us in our understanding of Peirce's work itself.

A major background theme in Burch's book is the notion of Peirce's "Unitary Logical Vision" (3 ff.).

According to this view (which is essentially correct), Peirce arrived at his basic view of logic quite early--

perhaps by the 1860's--and, over the years following, experimented with a large variety of logical

[1]Burch, Robert W., **A Peircean Reduction Thesis: The Foundations of Topological Logic** (= Philosophical Inquiries I), Lubbock, TX: Texas Tech Univeriess Press, 1991.
[2]A relation of adicity *n* is, essentially, an *n*-adic relation.

formalisms, each of which was an expression of the same basic notion of logic--or, perhaps it is better to

say, was an expression *consistent with* that same basic notion of logic, since each of the expressions

emphasizes a different shade (sometimes more than just a shade) of the meaning of Peirce's logic.

Actually, in Peircean terms, **Logic** is an abstraction; in Peirce's own terms

> An abstraction is something denoted by a noun substantive, something having a name,
>
> whether it be a reality or whether it be a figment . . .
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> An abstraction is a substance whose being consists in the truth of some
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> propositions concerning a more primary substance (Peirce 1976 v. 4, 161).

the EXISTENCE of the abstraction is located on another level, the level which is **second** to the **third**

which is the abstraction.[3] The unifying general represented by this concept of logic is not, of course,

imaginary--it is a **real third**. In this case, the "existential instantiations" of this general are the various

notational systems, the *systems of diagrammatization* in which the logic is expressed. Peirce began his

mathematical work in logic with the algebras of Boole and DeMorgan, and made notable contributions to

these notations. Indeed, in some of his important contributions, we see how the alteration of the system of

diagrams permits us to gain new insights into the matter of logic; as Peirce remarked

. . . one must keep a bright lookout for unintended and unexpected changes thereby brought about in the

relations of different significant parts of the diagram to one another. Such operations upon diagrams,

whether external or imaginary, take the place of the experiments upon real things that one performs in

chemical and physical research. Chemists have ere now, I need not say, described experimentation as

The putting of questions to Nature. Just so, experiments upon diagrams are questions put to the Nature

of the relations concerned (CP 4.530).[4]

His overall work on logic moves this notion of mathematics as experiment on diagrams to another level,

on which the "unintended and unexpected changes" are not connected with insight into and proof of, say,

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[3]Note that we will find this definition of abstraction most relevant also when we come to the relationship between what Burch will call *Interpretations* and what he calls *Enterpreta-tions*. The relations (as relations *simpliciter*) on the level of the *Interpretation* will be connected with the existent seconds (relations as *n*-tuples of *relata*) on the level of the *En-terpretation*.

[4]References to Peirce 1936-1958 are as usual in Peirce scholarship. They will be prefixed by "**CP**," and give a volume and paragraph number; thus, "4.530" is paragraph 530 of volume 4.

specific theorems of mathematics, but with the whole structure of logic. An example comes with the role of quantification in his understanding of the connection between the *hypothetical* and the ***de inesse conditional***:

> In a paper which I published in 1880, I gave an imperfect account of the algebra of the copula. I there expressly mentioned the necessity of quan-tifying the possible case to which a conditional or independential proposi-tion refers. But having at that time no familiari-ty with the signs of quanti-fi-cation which I developed later, the bulk of the chapter treated of simple consequences *de inesse*. Professor Schröder accepts this first essay as a satisfactory treatment of hypotheticals; and assumes, quite contrary to *my* doctrine, that the possible cases considered in hypotheticals have no multi-tudinous universe. This takes away from hypothe-ticals their most charac-teristic feature (2.349).

Once he had discovered the quantifier, he is able to say that

> the quantified subject of a hypothetical proposition is a *pos-sibility*, or *possible case*, or *possible state of things*. In its primitive state, that which is *possible* is a hypothesis which in a given state of information is not known, and cannot certainly be inferred, to be false. The assumed state of information may be the actual state of the speaker, or it may be a state of greater or less information. Thus arise various kinds of possi-bility (2.347).

Although this last passage is from 1902, the insight dates from his earlier possession of the quantifier as a tool (see 3.374--1885--for example).

The role of the hypothetical as third to the second of the *de inesse* conditional is just one instance of emphasis on thirdness in the logical work of Peirce; thirds emerge in a variety of systematic ways in Peircean logic. One of the most important has to do with his use of notation. As Burch notes, Peirce's logical work proceeds with a unity of vision, but expresses that vision in a large variety of ways. In addition to the well-known work on algebraic logic and the Existential Graphs, there are a plethora of other systems with which he experiments, ranging from "Qualitative Logic" to "Streamer Notation" (Peirce 1976, v. 4, 106 ff.) to "Entitative Graphs"; in related developments, he experimented with what he called

"secundal notation" (*e.g.*, ms. 51-67, Robin) for number (this is binary notation--in some senses, 'secundalÿ is actually a better name than is binary).

But with all of the experiments in diagrammatization which Peirce undertook, the notation which he prefers as he nears the end of his life is that of the Existential Graphs. This preference involves

> . . . The purpose of the System of Existential Graphs, as it is stat-ed in the Prolegom-ena
> [**CP** 4.533], [which is] to afford a method (l) as simple as possible that is to say, with as
> small a number of arbitrary conventions as possi-ble), for repre-senting proposi-tions (2)
> as iconically, or diagrammatically and <3) as analytically as possible (**CP** 4.651, n1).

He felt that this purpose was accomplished by **EG**, at least as far as he had been able to take this system of notation. One of the key areas in which the Existential Graphs accomplished this purpose, for Peirce, was in their representation of *continuity*--or, perhaps better, of logical material of which continuity is an important aspect:

> The line of identity . . . very explicitly represents Identity to belong to the genus Continuity
> and to the species Linear Continuity. But of what variety of Linear Continuity is the heavy
> line more especially the Icon in the System of Existential Graphs? In order to ascertain
> this, let us contrast the Iconicity of the line with that of the surface of the Phemic Sheet.
> The continuity of this surface being two-dimension-al, and so polyadic should represent
> an external conti-nuity, and especially, a continu-ity of experien-tial appearance.
> Moreover, the Phemic Sheet iconizes the Universe of Discourse since it more
> immediately represents a field of Thought, or Mental Experi-ence, which is itself directed
> to the Universe of Discourse, and considered as a sign, denotes that Universe. Moreover,
> it [is because it must be understood] as being direct-ed to that Universe, that it is iconized
> by the Phemic Sheet. So, on the principle that logicians call "the Nota notae" that the sign
> of anything, X, is itself a sign of the very same X, the Phemic Sheet, in representing the
> field of attention represents the general object of that attention, the Universe of
> Discourse. This being the case, the continuity of the Phemic Sheet in those places,
> where, nothing being scribed, no particular attention is paid, is the most appropriate Icon

possible of the continuity of the Universe of Discourse where it only receives general

attention as that Universe - that is to say of the continuity in experiential appearance of

the Universe, relatively to any objects represented as belonging to it.--From "The Bedrock

beneath Pragmaticism" (2) 1906 (**CP** 4.561, n1).

So thirdness--continuity--and its representation is central in **EG**, as it is in Peirce's thought overall.

Burch seeks to deal with continuities pertaining to the Peircean Universe of Discourse on yet

another level, the level of what he calls the "relation *simpliciter*"; this concept figures in his discussions of

semantics for his system of Peircean Algebraic Logic--**PAL**. Here he does something which may appear

curious, but which is consistent both with his treatment of Peirce and with the project of connecting this

material to the **FPR** tradition. In speaking of semantic interpretation of **PAL**, he introduces two notions of

interpretation, which he calls *Enterpretation* and *Interpretation*. He associates these notions respectively

with *Extensional* and *Intensional* treatments of semantics.

Now, speaking of extension *vs.* intension is, perhaps, not the happiest use of terminology. As I

have noted, I see Burch's work as, among other things, an essay at communication with the **FPR** tradition

(which speaks of this contrast); but this tradition doesn't think too highly of the "intensional," whatever that

may mean; whatever is worth talking about, it would seem, may be discussed in extensional terms (see,

e.g., Kneale, 604-5). The term "intensional" and its contrast with "extensional" is left vague and receives

very little attention. And we find that concepts which are characterized as "intensional" commonly become

susceptible to "extensional" interpretations--a prime example is the modal logic of Lewis and its

interpretation in the semantics first developed by Kripke and by Prior. And even more difficult "intensional"

concepts (such as that of the counterfactual conditional and other subjunctive constructions) may be

treated in systems such as the "neighborhood semantics" of Segerberg (1971). A wag might suggest that

one explication of "intensional" is "that which has not yet been sufficiently investigated to find an

'extensionalÿ interpreta-tion." But use of the contrast is hoary with tradition, going back to the *Port Royal

Logic*, anyway (Kneale, 318). I might suggest that the distinction is, at least often, one of perspective

rather than of "content." The "same" set of mathematical objects might be intensional from one

perspective, and extensional from another. But I will suggest further that the perspectival difference

involved is not merely a *nominal* distinction, but is (or can be) *real* . And just as the perspectival

differences are real, so too are the relations simpliciter which Burch works at describing mathematically.

So this is the kind of reading I give to Burch's carefully drawn distinction between *Enterpretations* and

Interpretations. If we take care in following Burch's use of the extensional-intensional contrast, then, the

terminology may have some value as a means of constructing bridges between Peirce and the **FPR**

tradition.

      Interpretations--which constitute the "intensional" aspect of this semantics, emphasize the

relations involved as **thirds**, while *Enterpretations*, the "extensional" aspect, emphasize the relations

involved as *n*-tuples of *relata*. I note that Kerr-Lawson has remarked that "Certainly, Peirce did not think of

relations as sets of *n*-tuples" (540). Actually, Peirce ***did*** think of relations in this way! Though not only in

this way. Although Peirce was unquestion-ably a realist so far as relations are concerned, he did not

exclude seconds from consideration. In fact, the categoreal interplay between secondness and thirdness

is an essential feature of his theory of abstraction, an abstraction being

      a substance whose being [which is secondness with respect to the thirdness which is figure in our

      view of the abstraction] consists in the truth of some propositions concerning a more primary

      substance (Peirce 1976, 161; see Zeman 1962 for detailed explication).

The relation is real, but its **existence** (which is not by any means all there is to say about this reality) is

found on the level of the domain of the relation, in the ***n***-tuples of *relata* which constitute this domain, and

which are second to the third of the relation itself. And this interpretation finds support in Peirce's own

discussions of the logic of relatives. In 1882, he remarks that

      A dual relative term such as "lover," "benefactor," "servant," is a common name signifying a pair

      of objects . . . (**CP** 3.328). A general relative may be conceived of as a logical aggregate of such

      individual relatives (**CP** 3.329).

So, far from Peirce's "not think[ing] of relations as sets of *n*-tuples," we see that he deserves credit as one

of the inventors of the contemporary notion of an *n*-adic relation as a set of *n*-ads!

      But as we note, there is more to Peirce's view of relation than this. Just as the view that a relation

is a set of *n*-tuples of relata corresponds to the *Enterpretations* of Burch, so does the view that a relation

is something real in addition to this correspond to his *Interpretations*. I think that a very important--perhaps the most important--contribution of Burch's analysis is his making explicit of these two perspectives on the semantics of relations.

In sifting through the large amount of material presented here on thirdness and relation, I note that Kerr-Lawson, in his discussion of Burch, suggests three "accounts of thirdness":

> A first option, following Burch's text, would be to emphasize predicates. Without ternary predicates . . . the logic attainable is weak and deficient . . .
>
> A second somewhat different approach seeks a stratagem which incorporates teridentity into the logical apparatus. With this done, the classical reduction result, saying that binary and unary predicates are sufficient, will be valid. Nonetheless, Peirces's point is not affected; thirdness is required, whether teridentity is placed among the given predicates or among the logical symbols.
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> A third possibility would be to shun any exact formulation in logic of thirdness, but to argue that the metaphysical principle of thirdness is *exemplified* by the presence of teridentity or something equivalent in logic. One should not, however, look to teridentity for a *definition* of thirdness. That logic requires thirdness is known in advance, in virtue of its using symbols to mediate among various other things. Peirce himself seems closer to this third position (Kerr-Lawson, 542-3).

Kerr-Lawson's analysis is very useful; I would emphasize, however, that these three accounts are by no means mutually exclusive, nor is Burch's approach limited to the "first option." In fact, Kerr-Lawson's analysis itself comes close to an authentic Peircean analysis based on the categories--if we "emphasize predicates" as a location for thirdness, we operate on a level prior to reflection (this would be firstness in this analysis); shifting attention to "incorporat-[ing] teridentity into the logical apparatus" would correspond to secondness, while Kerr-Lawson's "third possibility" corresponds to thirdness. So the three possibilities are not only compatible, but complementary. And Burch has not simply tried to locate thirdness on the level of predicates. His development of the concept of *Interpretation* as contrasted with *Enterpretation*

emphasizes this; as we have suggested, these two approaches to the semantics of PAL differ as third differs from second.

And something like Kerr-Lawson's analysis also will apply in Burch's treatment of hypostatic abstraction (Burch 105-16), which is yet another Peircean approach to thirdness. Burch analyzes hypostatic abstraction in terms of predicates; an

example is that of the triadic relation

$$x_1 \text{ gives } x_2 \text{ to } x_3$$

. . . Abstracting hypostatically on this relation consists of asserting that there is at least one entity of a special sort of entities; namely, instances or examples or acts of giving. We might also call these entities "givings" or instances of the obtaining of the triadic relation of giving. . . .

In quantificational logic, this hypostatic abstraction would be represent-ed by replacing the primitive well-formed formula

$$G(x_1, x_2, x_3)$$

with the will-formed formula

$$(\ddot{y}y)[\mathbf{G}(y) \text{ \& } I_1(x_1, y) \text{ \& } I_2(x_2, y) \text{ \& } I_3(x_3, y)] \quad ,$$

where $\mathbf{G}(y)$ stands for the monadic relation "y is a giving," $I_1(x_1, y)$ stands for the dyadic relation "x_1 is a donor with regard to y," $I_2(x_2, y)$ stands for the dyadic relation "x_2 is donated with regard to y," and $I_3(x_3, y)$ stands for the dyadic relation "x_3 is a recipient with regard to y" (Burch 106).

Although this makes it appear that the triadic G is reduced to the monadic **G** and the dyadic I_k, Burch argues that on his analysis "hypostatic abstraction involves *in every case* [concepts] all of which are definable in PAL only through the teridentity relation" (107). This locates the thirdness essential to hypostatic abstraction on the level of the predicate (the first level in Kerr-Lawson's account of thirdness). But the thirdness involved in hypostatic abstraction is also found on the other levels, even in Burch's account. The relationship between *Enterpreta-tions* and *Interpretations* is, perhaps, a place where this thirdness is found on Kerr-Lawson's second level--the real "relations *simpliciter*" of the *Interpretations* are

hypostatic abstractions whose **existence** or **actuality** is found, as we have noted, in the ordered *n*-tuples of the *Enterpretations*. And the level of thirdness in Kerr-Lawson's third level (which might be called "thirdness-as-background") is required for any understanding at all of hypostatic abstraction. I note that Burch's treatment of hypostatic abstraction as illuminated by Kerr-Lawson's analysis continues the communication between Peircean logic and the **FPR** tradition which is a major effect of this book.

In his preface to ***A Peircean Reduction Thesis***, Burch remarks that, while Peirce recognized that an *n*-adic relation is intimately associated with the class of *n*-ads,

> relations for Peirce were something *sui generis*; they were relations *as such*, relations *simpliciter*, relations *period*. Moreover, the *terms* of Peirce's logical algebras were intended to denote or otherwise express such relations *sim-pli-ci-ter*. And it is reasonable to believe that it was relations as such that the primary reduction thesis Peirce had in mind was about (ix).

Burch has done a nice job of presenting a mathematical perspective on "relations as such" in his *Interpretations*; the link between *Interpretations* and *Enterpretations* helps bridge the gap between the Peircean approach to logic and that of the **FPR** school. And as Burch asks, and begins to answer,

> What are these relations *simpliciter* that the terms of Peirce's algebras and the pictures of Peirce's graphs express? As far as the formalism of logic goes, the answer to this question will be presented in the sequent formalism of this work. A logical formalism does not of itself, however, answer metaphysical questions. Peirce's full answer to the question as to the nature of relations as such lies in his metaphysics and semeiotic (Ibid.).

This is correct; Burch, however has done a good job of presenting a mathematical perspective on some very important real thirds in the thought of Peirce. And apropos of the link I have pointed to with the thought of what I have called the **FPR** school, Burch, on noting that "Peirce was no nominalist, no blind adherent of extensionalist devotion" (Ibid.), asks

> May the committed extensionalist therefore turn aside in scorn at this point? I do not think so. For the notion of a relations as such, as the formalism of this work attempts

to show, can be explicated consistent with the extension-alism of standard, nominalistic

logic (Ibid.).

And such an explication is, perhaps, the major contribution of this little book.

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